PAPER

A finite-element analysis on the free vibration of Japanese drum wood barrels under material property uncertainty

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Abstract: This paper presents a finite-element analysis on the free vibration of Japanese drum wood barrels under insufficient material property data. Unlike isotropic material such as steel, wood behaves like an orthotropic composite material, whose elastodynamic characterization in a cylindrical shell needs Young's modulus in the longitudinal (in-grain) and circumferential (cross-grain) directions, shear modulus, and Poisson ratios. Due to measurement difficulty encountered during the process of testing, only the longitudinal Young's modulus and the specific gravity of the wood were measured. In the analysis, the finite-element models of the drum were constructed using conical shell elements. The required unknown elastic constants were estimated consecutively by a try-and-error approach, and the estimated values were reached when the computed resonance frequencies matched simultaneously with those of the seven lower modes measured in experiments. In order to accomplish this, both the estimated constants and the finite-element analysis must be within acceptable range of accuracy. It was found that the values of the circumferential (cross-grain) Young's modulus, which was unknown at the beginning of the study, turns out to be crucial in determining the lower mode resonance frequencies. The usefulness of this analysis is that the estimated elastic constants can now be used for updating the finite-element model. The updated model can then be used calculating the higher order modes which cannot be practically obtained through measurements.

Keywords: Finite-element, Modal analyses, Wood structures, Unknown material properties

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1. INTRODUCTION

A Japanese drum consists of a hollow wood barrel with two stretched cow skins (the diaphragms) at its ends. In analyses, the diaphragms are usually treated as two almost identical circular plates under high tension. The wood barrel is much more rigid and massive than the diaphragms. Previous study on the coupling between the two membranes of a Japanese drum through the air inside the wood barrel, concentrating on the resonance frequency changes due to the coupling, has been investigated using semi analytical method [1]. In that study, the wood barrel wall was treated rigid, and the membrane bending stiffness was neglected. It is believed that the elastic vibration of the wood barrel may have a significant influence on the overall sound production of a drum. To address this concern, this paper treats the drum barrel as a hollow elastic body and investigates numerically its free vibration regarding the resonant frequencies and modal structures. Figure 1 shows the wood barrel as a cylindrical shell of varying radii (r) and thicknesses (h), from one end to the other. It is noted that, in Fig. 1(b), the thickness and length of the drum are not drawn according to the actual proportion. Experiments for measuring resonance frequencies and mode shapes had been performed with two drums which have different interior surfaces [2]: one is smooth while the other wavy (see Fig. 1). There were no diaphragms attached to both of the drums. The type of wood used for the drum bodies is zelcova.

Analytically, a drum body is generated by rotating its cross-section shown in Fig. 1(b) with respect to a center axis, say the *z*-axis. In order to handle the varying radii and thicknesses of the cross-section, a finite-element analysis using conical shell elements will be used.

Most analyses are normally conducted with known material properties. This is especially true for metal structures where elastic properties are well known and

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(a) External geometry of the Japanese drum tested.



Fig. 1 Geometry of the Japanese drum tested.

can be easily found in literatures. As will be shown later, unlike metals, woods are nature examples of orthotropic composite materials, whose elastodynamic characterizations need Young's moduli in the longitudinal (in-grains) and the circumferential (cross-grain) directions, shear modulus, and Poisson ratios. These physical properties also depend largely on moisture content [3] and may change over time. The measurement of wood properties in all directions is difficult since thin, long and "straight" rectangular wood pieces except for the longitudinal direction are hard to get because they dry up quickly and change shapes. This is the reason why some of the above elastic constants were unable to measure in this study, and only the longitudinal Young's modulus and the specific gravity of the wood were measured. The other constants are unknown. Therefore, this study is not a straightforward numerical investigation with known material properties aiming mainly to verify the finite-element model with experimental data. The unknown constants must be obtained first. This paper thus focuses on how these unknown constants are estimated and how the estimated constants may be used for updating the finite-element model for other uses; such as for computing the higher order modes of the same drum which are not reported from measurements, or the effects on the drum vibration due to the changes of material properties, shapes and thickness distributions, etc. The building blocks of the finite-element model of the drum are the orthotropic conical shell elements, which will be described briefly as follows.



Fig. 2 Conical shell coordinates (local and global).

2. CONICAL SHELL FINITE ELEMENT

The geometry of a two node conical shell element of thickness (h) is shown in Fig. 2. In vibration analysis, very fine meshes are required because the wavelengths of the higher order modes are short. The distances between two nodes (L) are usually much smaller than the radii of the element. Figure 2 was drawn out of proportion in order to conveniently illustrate the geometric features of the element. The governing equations and procedures given by Skelton and James [4] were used in deriving the element stiffness and mass matrices. Equations (1)–(5) which were given in [4] are repeated here to illustrate how the stress and strain are related by the material constitutive matrix.

In common engineering practice, the time dependent displacement vector of an axisymmetric shell at its midsurface (half thickness point) at any location may be expressed as the following Fourier expansion,

$$U(z,\phi,r)e^{i\omega t} = \sum_{n=-\infty}^{\infty} U(z,n,r)e^{in\phi}e^{i\omega t}$$
(1)

where the time-dependent factor $e^{i\omega t}$ will be omitted hereafter to simplify mathematical expressions. In the absence of external forces, the free vibrations of an axisymmetric shell are uncoupled among the circular harmonics. Each circular harmonic, $e^{in\phi}$, can be treated independently. The summation over *n* in Eq. (1) will be unnecessary and be dropped hereon. Since there are two nodes in each element, say *i*- and *j*-node, we may define a local coordinate *s*, starting from the *i*-th node (z_i, r_i) such that $z = z_i + s \cos \beta$, $r = r_i + s \sin \beta$. When s = L, $z = z_i + L \cos \beta = z_j$, $r = r_i + L \sin \beta = r_j$, and *L* is the slant length between the two nodes. The displacement vector inside an element between *i*- and *j*-node can then be expressed in the local coordinates (s, ϕ) as

$$u(s,\phi) = u(n,s)e^{in\phi}$$
(2)

where $u(n, s) = \{u_s(n, s), u_\phi(n, s), u_N(n, s), \psi_s(n, s)\}; u_s(n, s)$ shown in Fig. 2 as u_{sl} , is the displacement in *s*-direction; $u_\phi(n, s)$ is the circumferential displacement; $u_N(n, s)$ shown in Fig. 2 as u_{Nl} , is the normal displacement; $\psi_s(n, s)$ is the slope of $u_N(n, s)$ in the direction of *s*-axis, i.e., $\psi_s = (\partial u_N)/\partial s$. In each circular harmonic, the relationship between the two coordinate systems may be expressed as

$$U(z, n, r) = [T]u(n, s)$$
(3)

where

$$[T] = \begin{bmatrix} \cos \beta & 0 & -\sin \beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \beta & 0 & \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is to note that the circumferential displacement and the two rotational displacements (the slope) are the same in both coordinates. The local coordinates will be used in the derivation of the element stiffness and inertial matrices. By using [T], these local stiffness and inertial matrices can be transformed to the equivalent matrices in the global coordinates.

In theory of elasticity, stresses $\{\sigma\}$ and strains $\{\epsilon\}$, and how they are related through the material constitutive matrix are essential for the development of the system dynamic equations. As shown in [4], the required strain components are, $\{\epsilon\} = \{\epsilon_s \ \epsilon_\phi \ \epsilon_{s\phi} \ \kappa_s \ \kappa_\phi \ \kappa_{s\phi}\}$, where ϵ_s and ϵ_{ϕ} are the mid-surface in-plane strains, $\epsilon_{s\phi}$ is the shear strain in the same (s, ϕ) plane, κ_s and κ_{ϕ} are the changes in curvature of the normal displacement with respect to s- and ϕ -axis, respectively; $\kappa_{s\phi}$ is the change in twist. The corresponding in-plane forces and bending moments (per unit length) are, $\{\sigma\} = \{N_s \ N_\phi \ N_{s\phi} \ M_s \ M_\phi \ M_{s\phi}\},$ in which N_s , N_{ϕ} , and $N_{s\phi}$ are the in-plane forces; M_s , M_{ϕ} , and $M_{s\phi}$ are the bending moments. For simplicity and avoiding the repeated use of awkward transpose symbols, both $\{\sigma\}$ and $\{\epsilon\}$ shown above were written as row vectors, but they will serve as column vectors as they must be related to the constitutive matrix [D] as shown below,

$$\{\sigma\} = [D]\{\epsilon\}.$$
 (4)

The strain components as functions of displacements in a conical shell can then be written, according to [4], as

$$\epsilon_s = \frac{\partial u_s}{\partial s}$$

$$\epsilon_\phi = \frac{1}{r} \frac{\partial u_\phi}{\partial \phi} + \frac{u_N \cos \beta + u_s \sin \beta}{r}$$

$$\epsilon_{s\phi} = \frac{1}{r} \frac{\partial u_s}{\partial \phi} + \frac{\partial u_\phi}{\partial s} - \frac{u_{\phi} \sin \beta}{r}$$

$$\kappa_{s} = -\frac{\partial^{2} u_{N}}{\partial s^{2}}$$

$$\kappa_{\phi} = -\frac{1}{r^{2}} \frac{\partial^{2} u_{N}}{\partial^{2} \phi} - \frac{\sin \beta}{r^{2}} \frac{\partial u_{N}}{\partial s} + \frac{\cos \beta}{r^{2}} \frac{\partial u_{\phi}}{\partial \phi}$$

$$\kappa_{s\phi} = -\frac{2}{r} \frac{\partial^{2} u_{N}}{\partial s \partial \phi} + \frac{2 \sin \beta}{r^{2}} \frac{\partial u_{N}}{\partial \phi} + \frac{2 \cos \beta}{r} \frac{\partial u_{\phi}}{\partial s}$$

$$-\frac{2 \sin \beta \cos \beta}{r^{2}} u_{\phi}$$
(5)

The stress $\{\sigma\}$ and strains $\{\epsilon\}$ vectors can be related by the following 6×6 orthotropic constitutive matrix, i.e.,

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 & 0 & 0 & 0 \\ D_{21} & D_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & D_{45} & 0 \\ 0 & 0 & 0 & D_{54} & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix}$$
(6)

where $D_{11} = E_s h/(1 - v_s v_{\phi}),$ $D_{22} = E_{\phi} h/(1 - v_s v_{\phi}),$ $D_{12} = E_s v_{\phi} h/(1 - v_s v_{\phi}),$ $D_{21} = E_{\phi} v_s h/(1 - v_s v_{\phi}),$ $D_{33} = G_{s\phi}h,$ $D_{44} = E_s h^3/12(1 - v_s v_{\phi}),$ $D_{45} = E_s v_{\phi} h^3/12(1 - v_s v_{\phi}),$ $D_{55} = E_{\phi} h^3/12(1 - v_s v_{\phi}),$ $D_{55} = E_{\phi} h^3/12(1 - v_s v_{\phi}),$ and $D_{66} = G_{s\phi} h^3/12.$

The elastic constants shown above are: E_s , the longitudinal Young's modulus; E_{ϕ} , the circumferential Young's modulus; $G_{s\phi}$, the shear modulus; ν_s and ν_{ϕ} , the Poisson ratios. It is noted that the relationship, $\nu_{\phi}E_s = \nu_s E_{\phi}$, must hold.

Based on the above stress-strain relationship, the mathematical expressions for the mass matrix [M(n)] and the stiffness matrix [K(n)] of each finite-element at the *n*th circular harmonic, are developed by following the detailed procedures given by a text book by Skelton and James [4]. In this study, those mathematical expressions in the global coordinates were evaluated by using the Matlab Symbolic Math package, and the closed-form algebraic solutions were obtained. Once each individual element matrices are computed, they are summed according to the standard finite-element procedures [5] to form the assembled system stiffness $[\mathcal{K}(n)]$ and mass $[\mathcal{M}(n)]$ matrices.

3. NUMERICAL RESULTS

The finite element model of the drum body consists of 116 equally spaced conical shell elements and therefore has a total of 468 degrees of freedom. In the drum with wavy shapes there are about eight elements per one half of a wavelength. This should have enough resolution to reflect the thickness changes of the drum body. The eigenvalues $(\lambda_i = \omega_i^2)$ and eigenvectors (U_i) for the free vibration of the drum body are determined by solving the following equation,

$$[[\mathcal{K}(n)] - \lambda_i[\mathcal{M}(n)]]\{U_i\} = 0.$$
⁽⁷⁾

All of the numerical results shown in this section were computed by using the Matlab routine, EIGR.

The wood properties of zelcova given from measurement are the Young's Modulus in the longitudinal direction, $E_s = 1.2 \times 10^{10}$ Pa, and the specific gravity = 780 kg/m³. The other required unknown elastic constants are Young's modulus in the circumferential direction (cross-grain), E_{ϕ} , shear modulus $G_{s\phi}$, Poisson ratios v_s and v_{ϕ} . At beginning of the try-and-error process, the modal resonance frequencies were computed with an educational guess of the values of the unknown constants. The computed frequencies were compared to the measured frequencies, and their differences were used as a guide for guessing the subsequent set of the values of those constants. After several iterations the best match between the calculated and the measured results were obtained by using $E_{\phi} = 1.7 \times 10^9 \text{ Pa}$, $G_{s\phi} = 1.36 \times 10^9 \text{ Pa}$, $v_s = 0.35$ and $v_{\phi} = 0.05$. It was found that the calculated resonance frequencies of the lower modes are most sensitive to the value of the cross-grain Young's modulus, E_{ϕ} .

The try-and-error process was also used for ensuring that the measured values of the wood property were unaffected by the possible changes of moisture content over time. Using slightly different values of E_s and specific gravity along with the estimated constants shown above didn't produce a better result. This shows that the measured values of the property were in fact unchanged and applicable.

3.1. Calculated and Measured Resonance Frequencies

The calculated mode shapes which will be shown in the next section are plotted as a rectangular product of the radial modal pattern as a function of z, $r_m(z)$, and the angular pattern in terms of circular harmonics, $\cos(n\phi)$, i.e., $r_m(z) \cos(n\phi)$. The following tables show the calculated and measured [2] resonance frequencies assembled in an ascending order and their corresponding order in $r_m(z)\cos(n\phi)$ designated as (m,n) modes. Since the drum body is not a uniform cylindrical shell and the boundary conditions are free at both ends, its eigen functions are complicated and cannot be expressed analytically. However, by inspecting the mode shapes calculated, $r_m(z)$ does behave roughly as a harmonic function, $\cos(m\pi z/L)$. In all the modes shown in the next section, the mode shapes behave roughly like $\cos(m\pi z/L)\cos(n\phi)$. This is the reason the mode order of m was set to begin at m = 0, which represents the case when the modal patterns are constant along the z axis. Therefore the lowest mode that corresponds to $r_m(z)\cos(n\phi)$ is designated as (0,2) mode. Numerical errors given below, in Tables 1 and 2, were determined by assuming that the measured frequencies were absolutely correct.

 Table 1
 Comparison of calculated and measured resonance frequencies (smooth shape).

Mode number	order (m, n)	Measured (Hz)	Calculated (Hz)	Error (%)
1	(0, 2)	137	136	-0.7
2	(1,2)	192	195	+1.5
3	(0,3)	383	381	-0.5
4	(1,3)	452	452	0
5	(0,4)	680	692	+1.8
6	(1,4)	761	768	+0.9
7	(1, 1)	800	802	+0.3

 Table 2
 Comparison of calculated and measured resonance frequencies (wavy shape).

Mode number	order (m, n)	Measured (Hz)	Calculated (Hz)	Error (%)
1	(0,2)	134	133	-0.7
2	(1,2)	185	189	+1.6
3	(0,3)	375	374	-0.3
4	(1,3)	435	438	+0.7
5	(0,4)	663	678	+0.8
6	(1,4)	735	744	+1.2
7	(1,1)	802	785	+2.1

The above tables show that for both of the smooth and wavy shapes, the calculated frequencies are within 2 percentages from the measured values. These errors would include the possible numerical errors and the uncertainty of the material properties. Because these errors are small enough, the estimated material properties obtained from the try-and-error approach ($E_{\phi} = 1.7 \times 10^9$ Pa, $G_{s\phi} = 1.36 \times$ 10^9 Pa, $v_s = 0.35$ and $v_{\phi} = 0.05$) are believed to be reasonable. The conical shell elements used in this study seem to be adequate for the first seven modes discussed above. However, for very high order modes, the element stiffness matrices may require to include the effects of transverse shearing, and perhaps, to use axisymmetric solid finite elements for more accuracy.

In order to help violinmakers to estimate the properties of violin top plate, Molin, Tinnsten, Wiklund and Jansson [6], applied orthotropic shell elements into a finite-element model to calculate vibration modes of wooden plates of non-uniform thickness. In order to make the calculated results in agreement with that measured, the material properties they obtained for a spruce plate were: $E_s =$ 1.55×10^{10} Pa, $E_{\phi} = 1.02 \times 10^9$ Pa, $G_{s\phi} = 0.75 \times 10^9$ Pa, and $\nu_{\phi} = 0.02$. These properties are different from those of zelcova obtained from this study, but believed to be within the normal spread among the woods.

3.2. Calculated and Measured Mode Shapes

The computed mode shapes which correspond to the



Fig. 3 Mode No. 1, the (m, n) = (0, 2) mode, resonance frequency = 133 Hz.



Fig. 4 Mode No. 2, the (m, n) = (1, 2) mode, resonance frequency = 189 Hz.



Fig. 5 Mode No. 3, the (m, n) = (0, 3) mode, resonance frequency = 374 Hz.

mode orders shown in Table 2 for the wavy drum body are shown in Figs. 3 to 9. It is noted that the picture on the right of each figure shows the angular pattern of the mode seen from the right at the right end point of the drum, not at its midpoint. The patterns of these lower order modes show



Fig. 6 Mode No. 4, the (m, n) = (1, 3) mode, resonance frequency = 438 Hz.



Fig. 7 Mode No. 5, the (m, n) = (0, 4) mode, resonance frequency = 678 Hz.



Fig. 8 Mode No. 6, the (m, n) = (1, 4) mode, resonance frequency = 744 Hz.

that the length scales of the modes are much longer than that of the waviness shown in Fig. 1(b). Therefore, the small perturbations in thickness variations due to waviness do not affect the overall smoothness of the calculated mode



Fig. 9 Mode No. 7, the (m, n) = (1, 1) mode, resonance frequency = 785 Hz.



Fig. 10 Measured mode No. 4 at 435 Hz.



Fig. 11 Measured mode No. 7 at 802 Hz.

shapes. Consequently, the mode shapes of the smooth drum body are about identical to the corresponding modes of the wavy drum body, except there are some differences in resonance frequencies.

For comparison, measured mode shapes of No. 4 and No. 7 are shown in Figs. 10 and 11, respectively [2]. Only two measured modes are shown because the similarities and differences between the corresponding computed and measured modes are common for all seven modes. Note that there is a small view angle difference and only the deformed mode shapes are shown in these figures. By visually comparing the computed and measured modal shapes, it can be said that the corresponding mode shapes are of the same kind. Large differences are observed at the right and left ends of the drum. It is authors' guess that these differences comes from the difficulties of experimental modal analysis of curved structures. The impact hammering excitation with the correct angle is not an

 Table 3
 Calculated higher order resonance frequencies (wavy and smooth shapes).

Mode number	order (m, n)	Wavy (Hz)	Smooth (Hz)	Diff. (Hz)
8	(2,0)**	843	865	22
9	(2, 1)	941	965	24
10	(1, 0)	976	984	8
11	$(2,0)^{**}$	999	1,011	12
12	(0,5)	1,008	1,036	28
13	(2, 2)	1,013	1,038	25
14	(1,5)	1,109	1,148	39

**Modes No. 8 and No. 11 are the breathing modes of similar shapes, whose longitudinal modal patterns are both symmetric with the midpoint of the drum. The modal displacements of Mode No. 8 are stronger near the midpoint while the modal displacements of Mode No. 11 are stronger near the both end points. The reason the resonance frequency of Mode No. 8 occurs at lower frequency than Mode No. 11 is because the wood barrel is thinner at midpoint and thicker at both ends (see Figs. 12 and 15).

easy task and three-dimensional accelerometers may have leakage among the perpendicular components. The important point is whether the corresponding modes are of the same kind. One should remind that measurement accuracy of resonance frequencies is easily achieved, which is also crucial in the present study.

3.3. Calculated Higher Order Resonance Frequencies and Mode Shapes

In the experimental modal analysis, accurate modal analysis becomes more and more difficult as the mode orders are getting higher and higher due to the modal overlaps among adjacent modes and decreasing signal to noise ratios of frequency response function measurements. In the test setup of this paper, modal parameters up to the seventh mode was comfortably measured. For higher order modes, the resonance frequencies and mode shapes can be computed by using the model updated with the material properties estimated from the earlier finite-element analysis in conjunction with the experimental data.

Table 3 shows the calculated resonance frequencies of the smooth and wavy drum bodies from the 8-th to 14-th orders. Figures 12 to 18 shows corresponding modal shapes to Table 3. These information may be useful for investigating the contribution of the drum body vibration to the sound radiated from the whole drum. The last column shows the resonance frequency differences between the smooth and the wavy shapes, which reflect the slightly higher frequencies of the smooth shape due to its slightly thicker body.

4. CONCLUDING REMARKS

In this study, a finite-element method using the conical shell elements for modeling the wood barrel is used. The



Fig. 12 Mode No. 8, the (m, n) = (2, 0) mode, resonance frequency = 843 Hz.



Fig. 13 Mode No. 9, the (m, n) = (2, 1) mode, resonance frequency = 941 Hz.



Fig. 14 Mode No. 10, the (m, n) = (1, 0) mode, resonance frequency = 976 Hz.

only known properties (from testing of a sample) of the wood, which the experimental drum model is made of, are the longitudinal Young's modulus and the specific gravity.



Fig. 15 Mode No. 11, the (m, n) = (2, 0) mode, resonance frequency = 999 Hz.



Fig. 16 Mode No. 12, the (m,n) = (0,5) mode, resonance frequency = 1,008 Hz.



Fig. 17 Mode No. 13, the (m, n) = (2, 2) mode, resonance frequency = 1,013 Hz.

By estimating all other required elastic constants in a tryand-error approach, a set of elastic constant of the wood which yielded the calculated resonant frequencies (and



Fig. 18 Mode No. 14, the (m, n) = (1, 5) mode, resonance frequency = 1,109 Hz.

mode shapes) in good agreement with experimental results were obtained. The elastic constants obtained in this study for zelcova are: $E_s = 1.2 \times 10^{10}$ Pa, $E_{\phi} = 1.7 \times 10^9$ Pa, $G_{s\phi} = 1.36 \times 10^9$ Pa, $v_s = 0.35$ and $v_{\phi} = 0.05$. Using these values the computed resonance frequencies matched well simultaneously with those of the seven lower modes measured in experiments. In order to accomplish this, both the estimated constants and the finite-element model must be within acceptable range of accuracy.

The experimental modal analyses had been performed for the two drums, but they are limited to only a small number of lower modes. The usefulness of this approach is that the estimated unknown properties can be used for updating the finite-element model, and from which the unmeasured higher order modes may be computed. This will be particularly useful for understanding the higher frequency modal characteristics of a drum wood barrel. The updated finite-element model may also be useful for drum designers to predict the effects on the changes of modal characteristics resulting from the changes of drum shape or thickness distribution.

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